The colours of quarks as new degrees of freedom

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Abstract

The origin of the colours of quarks has been explored and the number of colours equal to three has been derived from the fractal properties suggested in the statistical model.The quark gluon coupling constant has been reproduced and the properties of the intrinsic electric charges of quarks have also been studied.

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1.Introduction

It is now accepted that quarks fundamental constituents of matter,possess a new and hitherto unknown internal degrees of freedom called colour,to resolve several difficulties in the quark model.The colour quantum number is hidden as all known hadrons are colour singlets and it is visible only when it is probed at a momentum transfer which allows us to resolve the individual quark constituents.

The statistical model of the hadron as a quarkonium system,proposed almost three decades ago[1],has been widely used to study the properties of hadrons.The model,however,has undergone modifications along with its various ramifications and applications since its very inception[2].The power-law scaling,the fractal properties as well as the universality of hadrons have been analysed recently in the framework of the model[3].The scaling exponent in the production formula of hadrons suggested by the model,has been found to be in conformity with the recent experimental findings.The colour factor ratio[4] has been derived recently from the fractal properties of hadrons and is found to be in exact agreement with the corresponding QCD value.In the current investigation we have attempted to trace the origin of the colour of quarks in the framework of the model.The real and virtual quarks in the sea of quark-antiquark ($q\bar{q}$) pairs in the model have been assumed to be identical and indistinguishable from one another and we arrive at the number density of quarks $n_q(r)$ inside the hadron.This has led us to derive the number of colours N_c of quarks equal to 3,as new degrees of freedom.As a result, the quark gluon coupling constant has also been derived and is found to be in exact agreement with the corresponding QCD

estimate.

Using the theoretically derived value of $N_c = 3$ as an input, it has been possible to explore also the origin of the fractional electric charges of quarks of different flavours and to obtain the bounds of their absolute magnitudes. On the other hand,it was suggested earlier[5] that the charges of the u and d quarks have to be known from some external source in order to infer the numerical value of N_c.

2. Colours as new degrees of freedom

Due to the identity and indistinguishability of the real and virtual quarks in the sea as described in the model, a special kind of geometric complexity emerges and the hadron turns out to be a fractal object.The local density of quarks(or anti-quarks)has been found to depend only on the size parameter r_0 of the hadron as [1-4]

$$n_q(r) = f(r_o)g(\xi) \tag{1}$$

for $r \leq r_0$ and $n_q(r) = 4$ for $r \geq r_0$. Here $\xi = r_0 - r$, $f(r_0) = \frac{315}{64\pi r_0^{9/2}}$ and $g(\xi) = \xi^d$ where $d = 3/2$. The fractal curve represented by $g(\xi)$ correspond to the region bounded by the interface and using the polygonal approximation to cover the fractal,we come across the minimum number of line segments,each of measure ϵ^d with a small interval of length ϵ.Thus $g(\xi)$ represents a d-dimensional curve embedded into the $D = 9/2$ dimensional space and $d = 3/2$ appearing as an exponent in defining the measure.Again,the spin degeneracy factor,arising from the two allowed values of the spin quantum number is 2 as the quark is a spin 1/2 particle.Consequently, the resulting factor turns out to be equal to 2d=3.This new factor appears

as the internal degrees of freedom to be assigned to the quark and has been named as the colour degrees of freedom i.e. $N_c = 2d = 3$.

3. Applications

The colour factors C_F and C_A are known to determine the intensity of gluon radiation off a quark and a gluon respectively and in the case of $SU(3)$ for QCD[6,7,8], one comes across the probability for a gluon to split into two gluons to be proportional to $C_A = N_c = 3$ corresponding to the three colours[5].On the other hand,using the numerical value of C_A/C_F derived previously by us in the framework of the model [4] in conjunction with the derived value of $N_c = 3$ in the current work,we get directly the value of $C_F = 4/3$, the quark gluon coupling constant and it agrees with the corresponding QCD estimate.

The Drell ratio $R \equiv \sigma(e^+e^- \rightarrow hadrons)/4\pi\alpha^2/s$ where $\alpha = e^2/4\pi$[6] can now be recast directly as

$$R = 3\Sigma_{i=1}^{N_f} e_i^2 \tag{2}$$

Here e_i is the intrinsic electric charge of the ith quark and N_f,the number of quark flavours which may contribute to the process and is constrained by $2m_{N_f} < \sqrt{s}$ where s is the centre of mass total energy squared of the e^+e^- system and m_{N_f} is the mass of the quark of flavour f.For the energy region $\sqrt{s} < 3$ Gev,only the u,d and s quarks contribute since this region is below the c-quark production threshold.The value $R \simeq 2$ is apparent below the threshold for producing charmed particles at energy $\simeq 3.7 Gev$[6].

Therefore,

$R = \frac{2N_c}{3} = 2$ we get

$$2 = 3(e_u^2 + e_d^2 + e_s^2) \tag{3}$$

for $E_{cm} > 2Gev > 2m_u,\ 2m_d,\ 2m_s$[7]. Hence

$$\frac{2}{3} = e_u^2 + e_d^2 + e_s^2 \tag{4}$$

Therefore,without any further input of experimental data,we get the sum of the squares of the electric charges of quarks of flavours u,d and s,equal to 2/3.Each term on the right hand side of (4) is therefore a proper fraction.In other words,the absolute magnitude of the charge of each quark corresponding to the three flavours is less than 1.Using $N_c^2 = 9$ corresponding to our derived value of $N_c = 3$ the decay width Γ for the $\pi^0 \to 2\gamma$ process may now be directly recast as

$$\Gamma = \frac{9({e_u}^2 - {e_d}^2)^2\alpha^2 \cdot m_\pi^3}{64\pi^2 f_{\pi^2}} \tag{5}$$

where $f_\pi = 91Mev$, the pion decay constant for $\pi \to \mu\nu$ decay.Using the experimental value of $\Gamma \simeq 7.57ev$ as the input,we get

$${e_u}^2 - {e_d}^2 \simeq 1/3 \tag{6}$$

Combining (5) and (6),we get

$$e_s^2 = 1 - 2e_u^2 \tag{7}$$

$$e_s^2 < 1 \quad \text{or,} \mid e_s \mid < 1 \tag{8}$$

Hence

$$2e_u^2 < 1 \quad \text{i.e.} \quad e_u^2 < 1/2 \quad \text{or,} \quad \mid e_s \mid < 1/\sqrt{2} \tag{9}$$

Therefore,we get from(6),

$$e_d^2 < 1/6 \quad \text{or,} \mid e_d \mid < 1/\sqrt{6} \tag{10}$$

These bounds on the intrinsic electric charges of quarks are obtained without any further experimental data as inputs.For the region $4Gev < \sqrt{s} < 9Gev$,there is an additional contribution from the c quark and the corresponding experimental value of $R \simeq 10/3$.Again, above the threshold for all five quark flavours i.e. for $\sqrt{s} > 10Gev$, $R_{exp} \simeq 1/3$.Using these two observed values of R for the two regions,in conjunction with(2),we arrive at $e_c^2 = 4/9$ i.e. $|e_c| = 2/3$ and $e_b^2 = 1/9$ i.e.$|e_b| = 1/3$ .Therefore,using $N_c = 9$,the fractional nature of the intrinsic electric charges of quarks follows as natural consequence and the absolute magnitudes of the charges of c and b quarks are explicitly determined.

4.Dimensions and conclusions

The origin of the colour degree of freedom of quarks is attributed to the fractal properties assigned by the model.The number of colours N_c has been derived theoretically to be equal to 3 through its dependence on $d = 3/2$ and the spin projection.Further,interesting bounds on the absolute magnitude of the intrinsic charges are obtained,which in turn suggest their fractional values.

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